inštitút finančnej politiky

Ministerstvo financií SR / www.finance.gov.sk/ifp

Stochastic forecast of the Slovak public debt

February 2016

Manual

Abstract

In this work we present a device for stochastic forecasting of the public debt that helps us achieve two objectives. First, it provides an independent, purely model-based, forecast of the public debt that cross-checks the official forecast. And second, estimated probability distribution allows us to quantify uncertainty of the forecast. At the core of the model, the standard debt accumulation equation is used to generate a large number of random forecasts. These are then used to extract the alternative – median – forecast along with confidence intervals for the central forecast. The debt equation summarizes stochastic behaviour of debt determinants – growth of nominal GDP, interest rates, primary surplus and exogenous financing of the debt – and is estimated in three steps. In the first step, growth of nominal GDP and interest rates are obtained in a VAR model. Independently in the second step we deal with the exogenous financing component. Finally, in the third step we utilise the outcome of the two preceding steps and simulate the fiscal reaction function together with the debt accumulation equation to generate a set of random forecasts of the debt.

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Acknowledgement

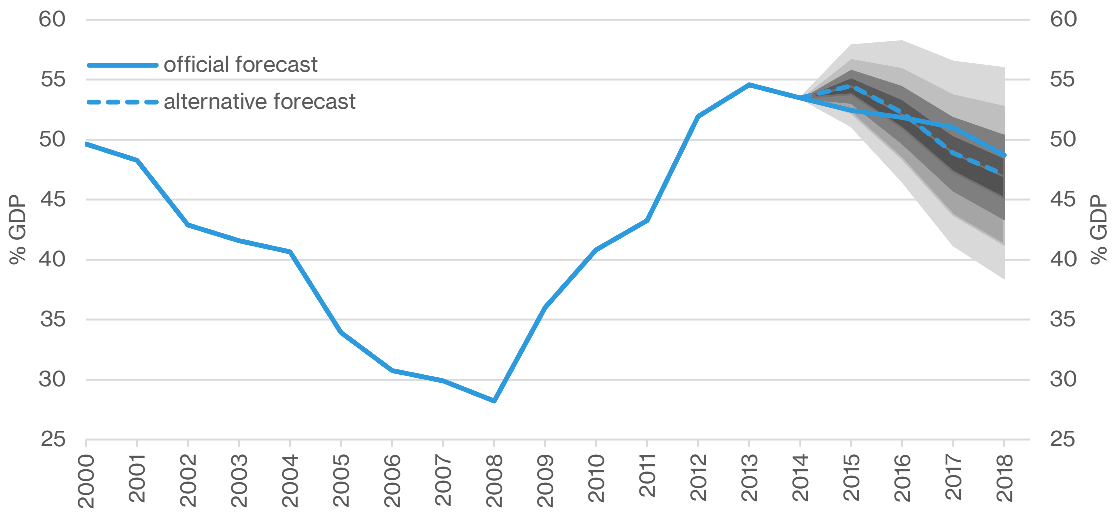
I would like to thank the reviewers from National Bank of Slovakia and Council for Budget Responsibility for their comments and discussion. Roman Vasiľ and Juraj Šuchta provided invaluable assistance with the data. I also benefited from comments from Eduard Hagara. All remaining errors are solely those of the author.

Table of contents

Introduction

The official forecast of the Ministry of Finance (October 2015) assumes that the level of public debt relative to GDP will monotonically decrease from 53.6% in 2014 to less than 49% by the end of 2018. The main drivers of the decline are lower interest payments, a robust growth of GDP and diminishing budget deficits. Obviously, there are risks to assumptions that the forecast is based on and the actual path of the public debt may be different. For this reason we graphically present uncertainty of the public debt forecast by using a fan-chart type graph in Figure 1. The grey shaded areas reveal that the probability that public debt will decrease below its current value in the medium-term is nearly 80%. The trajectory of the alternative forecast, which is the median forecast of the stochastic model described below, is in line with the official forecast when the maximum difference between the two trajectories reaches about 2 p.p. over the entire forecast horizon. Obviously, there are limitations to this purely statistical approach. Public debt is a policy variable and consequently may behave differently from its usual historical pattern. The fan-chart takes into account linkages between a set of key debt factors identified in the historical data and as such is unable to assess situations when certain policy actions aimed at managing public debt are more likely than others. Nevertheless the model is reasonably successful in explaining behaviour of the debt in the past.

Figure 1: forecast of public debt



Source: IFP

The text below shortly describes the possibilities that are available to deliver a model-based alternative debt forecast along with an estimate of uncertainty surrounding the forecast. After that we present the underlying method behind the fan-chart graph in a greater detail. In the last section we present more detailed results.

1. Available methods

The approaches to stochastic forecasting of public debt focus on the following debt accumulation equation:

$$d_t = \frac{1+i_t}{(1+g_t)(1+p_t)} d_{t-1} - ps_t + nondeffin_t. \qquad (1)$$

The debt equation specifies that the level of debt at a given year, d_t, depends on its own past value, d_{t-1}, interest payment at the rate, i_t, primary surplus, ps_t, the value of non-deficit financing, $nondeffin_t$, and finally the growth of nominal GDP, $(1+g_t)(1+p_t)$.

These methods rely on estimating interactions between the components determining the level of debt. Cherif et al. (2012), for example, estimate a VAR model consisting of all the debt equation components[1]. They then use the estimated model to simulate a large number of random forecasts of these components and consequently, by using the debt equation, forecasts of the debt. An advantage of this approach is that the VAR model endogenously captures interactions between all components of the debt equation. On the other hand, the model includes fiscal variables that are available on the annual frequency, which considerably limits the number of observations available for estimation. To utilise as much information as possible Medeiros (2012) follows a somewhat more complicated approach. In the first step he estimates and simulates a VAR model consisting of macro variables (interest rates, GDP, price of GDP), which are available on the quarterly frequency. Random forecasts of nominal GDP and interest rates are the input into the second step in which he simultaneously calculates random forecast of primary surplus and the debt. Primary surplus is an outcome of an estimated fiscal reaction function while the debt is an outcome of the above debt equation. Exact details of this method will become clear below.

The most appealing feature of the approach just described (apart from quantification of forecast uncertainty) is that it does not rely on any official forecast. Instead it provides an independent, purely model-based, outcome and can thus serve as a transparency enhancing device in communication of public policy. For this reason and longer quarterly time series we decided to closely follow the approach of Medeiros (2012). Below we go through all the details of the method and also provide some intermediate results.

2. The method

For sake of greater transparency we opt for estimating a model independent of the tools involved in the formal forecasting process. The model provides us with an independent central forecast of public debt together with quantification of uncertainty surrounding the projection. The underlying method is based on simulating a large number of random forecasts of the main determinants of debt which allow us

[1] In fact they exclude the component of non-deficit financing.

to calculate a large number of random forecasts of public debt through the debt evolution equation. When having a set of random forecasts of debt we can calculate the central (median) forecast, which we dub the alternative forecast. From the histograms of these forecasts we calculate the confidence intervals and present them by using the fan-chart graph above. Below we describe how we obtain random forecasts of all necessary ingredients and subsequently the forecasts of the debt.

2.1 The method: Step 1

As briefly outlined above, the method consists of three steps[2]. In the first step we estimate a small open economy VAR model on quarterly data. A possibility of employing the quarterly data helps a lot when the data sample is considerably short. The downside is that we cannot use fiscal variables relevant for the evolution of public debt. We deal with this issue in step three. Since Slovakia is a small and very open economy it is necessary to include an open economy dimension into the model. Therefore the set of endogenous variables include growth of domestic real GDP, inflation in price of domestic GDP, interest rates on domestic government bonds[3], nominal effective exchange rate[4], growth of foreign real GDP, inflation in price of foreign GDP, interest rates on foreign government bonds. The foreign variables are aggregates of the European Union member countries, which include our most important trading partners. The data sample covers the period 1997Q1 – 2015Q2 and where necessary the time series are seasonally adjusted.

Furthermore, to fulfill the assumption that Slovakia is a small economy and does not affect the foreign economy, we restrict the model appropriately. Apart from this restriction we do not impose any other restriction on the model. The model thus takes the following form

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + u_t \qquad (2)$$

where $Y_t = [Y_t^{domestic}, Y_t^{foreign}]'$ is a vector consisting of seven variables stated above, u_t is a vector of residuals and A_i is a matrix of parameters such that

$$A_i = \begin{bmatrix} 0 & A_{12}^i \\ A_{21}^i & A_{22}^i \end{bmatrix}.$$

In order to simulate random forecasts of the endogenous variables we need to estimate[5] the model and retrieve the variance-covariance matrix of residuals, $\Sigma = E(u_t u_t')$. Usually, one makes an assumption that the residuals are normally distributed with mean equal to zero and the covariance matrix Σ. Drawing the values of residuals from the normal distribution and iterating the estimated VAR model to produce

[2] Two steps when one ignores the contribution of the non-deficit financing component.

[3] The time series is an unweighted average of yields on 1 year and 10 year government bonds. This gives an average maturity of portfolio equal to 5.5 years which slightly overstates the actual average maturity of debt portfolio. In recent years the average maturity has raised to around seven years.

[4] Nominal effective exchange rate is a weighted average of domestic exchange rate with respect to exchange rates of our 15 most important trading partners in terms of volume traded.

[5] Based on the Schwarz information criterion the order of the model is set at 3.

forecasts of Y_t conditional on u_t and past values of Y is straightforward. However, it rarely happens that the residuals pass the test for normality[6]. This is also the case of our model. The problem is that the available sample does not appropriately represent the population hence the estimated statistics are not correctly recovered. For this reason we employ the wild bootstrapping method to resample the data from the given observed data. The wild bootstrap is also capable of handling the heteroscedasticity problem in case this problem applies to the sample of residuals.[7] In Figure 6 we present cross-sections of the debt forecast based on the Monte Carlo simulation exercise assuming normal distribution for the VAR residuals. Visually the differences are negligible.

Given the fact that recently the yields on government bonds have been quite close to zero, it may happen that some random forecast of interest rates slide to negative figures. In such a case we simply drop these forecasts from the set of resulting simulations[8].

We use the VAR model to simulate a set of forecast at the quarterly frequency. The debt equation, however, is specified at the annual frequency. Hence before we move on to the next step we need to convert the quarterly forecast data to annual data. The sum of quarterly GDP volumes is the annual value of GDP, average quarterly increases of GDP deflator is the annual GDP inflation, and the end of year government bond yields represent the annual interest rates that we need. Finally there is one more issue that we need to deal with. The implicit interest rate that is a part of the debt equation cannot be directly linked to the government bond yields which appear in the VAR model. The implicit interest rates are determined by past government bond yields of different maturity rather than newly issued bonds. Therefore we assume that the implicit interest rate on the forecast horizon is a weighted average of past implicit rate and yields on newly issued bonds, the outcome of the VAR simulation process, where the weight is the average maturity of debt

$$i_t = i_{t-1} * (1 - \frac{1}{am_t}) + \frac{1}{am_t} * by_t \qquad (3)$$

where i_t is the implicit interest rate on public debt as appears in the debt equation, by_t is the random forecast of yields on newly issued government bonds from the VAR model, am_t is the average maturity of public debt at the given time[9]. We use the issuance plan to calculate the (deterministic) path of the average maturity of debt portfolio on the forecast horizon. This completes the stochastic forecasts of nominal GDP and implicit interest rates.

[6] We also test the residuals for other potential sources of problems. We cannot reject the null hypothesis of no autocorrelation in the residuals in a Portmanteau test. Similarly we cannot reject the null hypothesis of no heteroskedastic residuals in the White test. Therefore OLS estimates should be an appropriate estimation method. The only obstacle preventing us from using a normal distribution random generator to simulate the future paths of model residuals is lack of evidence for normality. We can, however, deal with this issue by using a bootstrapping method.

[7] See Davidson, Monticini (2014)

[8] This is clear from the histogram graphs in Appendix 3.

[9] See Figure 3.

2.2 The method: Step 2

In the preceding section we elaborated on how to obtain random forecasts of macro ingredients of the debt equation. The next component to be treated, non-deficit financing of debt, is somewhat specific. It mainly reflects effects of one-off measures, such as privatization revenues used to repay a part of debt in the beginning of 2000s. For this reason it is hard to predict the behavior of this component and it is a common practice in the literature to drop it from the analysis. However, Figure 4 reveals that this component has contributed to the growth of public debt substantially in the past hence it is a rather important factor in assessing forecast uncertainty. Therefore we assume that this component contributes to uncertainty of the projected path of debt. We can safely reject the hypothesis that there is persistence in this series and model this components as an i.i.d. process. Due to this it does not affect (much) the median forecast of debt since its average value is close to zero and the median forecast of such a process is very close to its long-term average. The size of the standard deviation of this series is primarily influenced by the events in the first half of the sample. If we expect that the size of the component will be more in line with the size of the events in the second half of the sample, the standard deviation calculated using the entire sample may somewhat overestimate uncertainty of projections.

2.3 The method: Step 3

In the previous two steps we generated a large number of random projections of the following debt determinants: growth of real GDP, inflation in the price of GDP, interest rates and non-deficit financing component. The last thing we need is a set of forecasts of the primary deficit. We do this in an estimated fiscal reaction function of the form

$$ps_t = a_0 + a_1 * d_{t-1} + a_2 * gap_t + \mathcal{E}_t \qquad (4)$$

where, as before, ps_t is the primary surplus, d_{t-1} is the level of debt in the past period and gap_t is the output gap. It is rather difficult to reasonably estimate the parameters of the reaction function and for this reason we rely on estimates taken from Medeiros (2012) who uses panel data techniques, which are deemed to be more reliable in these problems. The output gap forecasts are taken from the first step above by applying the HP filter to GDP forecasts. The error term is assumed to be an autoregressive process of order one with an innovation process e_t[10]

$$\mathcal{E}_t = a_3 * \mathcal{E}_{t-1} + e_t. \qquad (5)$$

Compared to the original model we slightly changed the parameters. We keep the structural parameters (i.e. sensitivity of primary balance to debt, $a_1 = 0.078$, and business cycle, $a_2 = 0.691$) unchanged. We did a minor change to the constant term, $a_0 = -5.83$ such that the average value of the error term, e_t, is zero. Finally and more importantly we re-estimate the residual autoregressive equation (5). The coefficient on the lagged term, a_3, equals 0.43 while the standard deviation of the error term, e_t,

[10] For further details see Medeiros (2012).

is 2.20[11]. Our sample is rather short (consists of 17 observations) and formal statistical tests do not have enough power to deliver conclusive results[12]. Yet, allowing the residual term in (4) to follow an autoregressive process is likely to treat the serial correlation issue and even the LM test for serial correlation does not reject the hypothesis of no serial correlation in residuals, e_t. Furthermore, we also cannot reject the null hypothesis of homoskedastic errors. Finally and not surprisingly, we do not have enough evidence that the errors are normally distributed and for this reason we opt for a bootstrap approach in generating the forecasts of the primary balance.

The process of primary surplus forecast starts with calculating the value of the error term, $\mathcal{E}_t$, by using its autoregressive specification and randomly drawn innovation, e_t, by a bootstrap method. This piece of information together with the contemporary value of the output gap and past value of the public debt provides us with the contemporary value of primary surplus, ps_t. This is the last ingredient that we need for calculating the contemporary value of public debt, d_t through the debt equation (1). By iterating this procedure for all years of forecast we arrive at the full forecast of public debt along with primary surplus. The following scheme summarizes the iteration process of producing a random forecast of debt.

i) draw bootstrapped e_t and calculate $\mathcal{E}_t = a_3 * \mathcal{E}_{t-1} + e_t$
ii) calculate the value of primary surplus by using equation (4)
iii) calculate the value of public debt by suing equation (1)
iv) do this for each year of the forecast horizon

When we run this algorithm for each simulation of step 1 and step 2 we obtain the whole set of simulations of public debt[13]. Then from the set we extract necessary information to construct the fan-chart graph. Details on the construction of the fan-chart graph together with a broader set of results is presented in the next section.

3. Results in greater detail

The median forecast of all available random forecasts of debt is presented under the title alternative forecast in Figure 1. The mean forecast, which may slightly differ from the median forecast, sits in the center of the darkest grey interval. In order to construct the fan-chart around the mean forecast we need to order the forecasts increasingly from the simulation that predicts the lowest level of debt to the one that predicts the highest level of debt. The borders of the darkest interval in the center is determined by the two forecasts such that there are 10% of all ordered forecasts between them and the mean forecast. The width of every other interval is similarly given by two forecasts such that 10% of all realized forecasts lie between the borders

[11] Quality of the model can be judged from figure 10 in Appendix 3 which presents actual and fitted values of the primary deficit.

[12] A good example is the test for significance of the autoregressive parameter, a_3, in (5). Its point estimate is 0.36, which is rather far from zero, yet is statistically insignificant. This suggests that the term $\mathcal{E}_t$ may be treated as a serially not correlated component. Nevertheless, we allow this component to follow an AR(1) process which treats the autocorrelation issue in the error term $\mathcal{E}_t$.

[13] We draw 5000 random simulations in total.

of the interval. That means that the entire shaded area encompasses 80% of all random forecast centered at the mean forecast. The width of individual intervals increases in time reflecting growing uncertainty on longer horizons, which can be seen from the cross-sections of the fan-chart in Figure 5.

In order to see more into the sources of uncertainty we report cross-sections of the debt determinants in the first year of forecast and also in the fourth year of forecast. Two most volatile components projected one year ahead are primary surplus and the exogenous financing. Given its persistent nature, the histogram of implicit interest rate is rather narrow. Uncertainty of the nominal GDP growth is not too large too. Given the current low interest rates on government bonds it is likely that the implicit interest rate on debt will decrease in the medium-term. Moreover the growth of nominal GDP is also likely to stabilize and these two factors should contribute to lower primary deficits. This implies that all determinants are likely to contribute to decreasing trajectory of debt. Uncertainty about the future paths of nominal GDP and interest rates are expected to increase in the medium-term and this determines growing uncertainty of the debt forecast. Prospects for the primary surplus is also more uncertain into the future. On the other hand, uncertainty of the exogenous financing component remains about the same. All in all, the debt is supposed to decrease due to all main factors that determine the level of debt. The probability that debt will decrease in the medium-term is about 80%. Furthermore, the probability that debt will be below the official forecast is more than 60%.

Both the official and alternative forecasts decline on the projection horizon. However, factors driving them somewhat differ. Interest rates, GDP growth and primary deficit will contribute to the decreasing path of the official forecast. On the other hand, the alternative projection expects that mainly the interest rates and GDP growth will contribute to the declining path of debt. Figure 11 presents these findings in greater detail. Figure 12 summarizes the contributions of individual factors that drive the deviation between the two trajectories. The interest rates and GDP growth push the path of the alternative forecast below the path of the official forecast while primary deficit does the opposite. The former effect, however, dominates the latter one hence the entire trajectory of the alternative forecast (except in 2015) is below the trajectory of the official forecast.

In Figure 8 we also report a fan-chart under the assumption that the exogenous, non-deficit financing, component of the debt equation will not contribute to overall uncertainty. It turns out that the median alternative forecast does not change much as we indicated above. The total width of the fan-chart in the first year of forecast is considerably narrower, which suggests that the exogenous component of debt financing plays a significant role in the short-run. On the longer horizon uncertainty about the remaining components takes over the main role as the shape of the fan-chart is rather similar to the one in Figure 1. Finally, in order to assess overall performance of the model we run an in-sample forecast exercise. In each year since 2002 we simulate a large number of stochastic forecasts assuming that we know the historical data, but do not have any actual data available at the start of the forecast. From the set of stochastic forecasts in every year we extract the median forecast and present it in Figure 7 along with the actual trajectory of the public debt.

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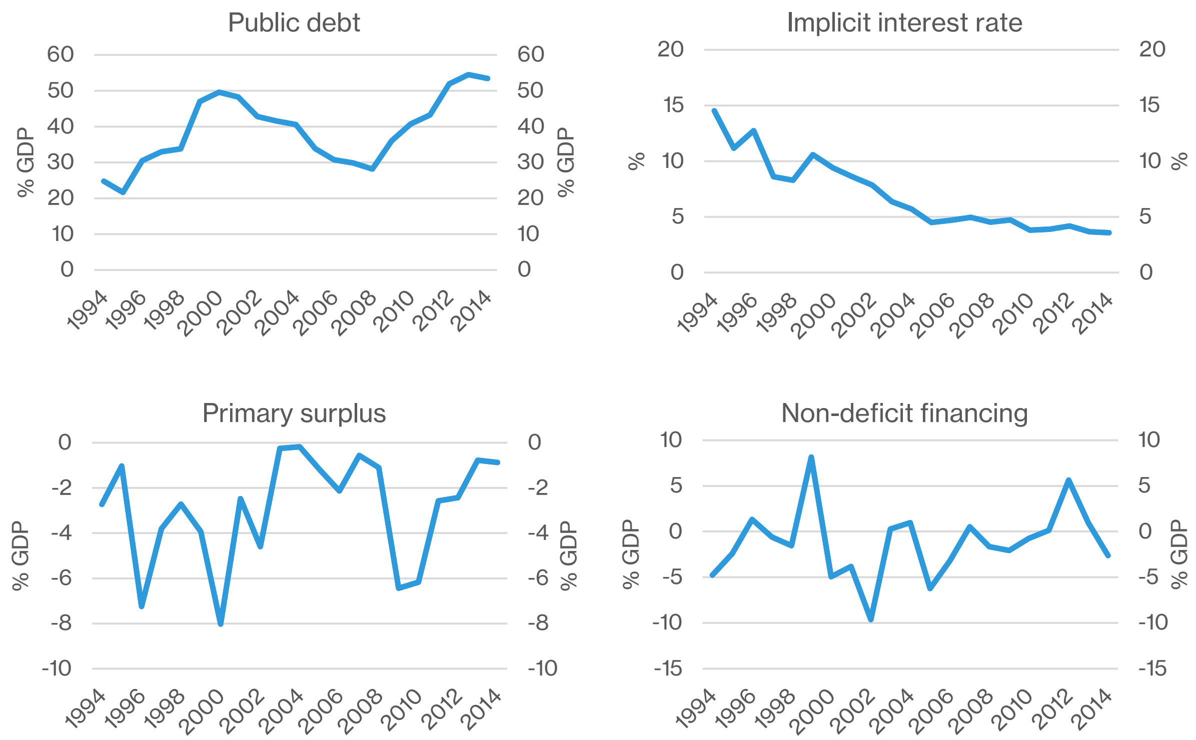
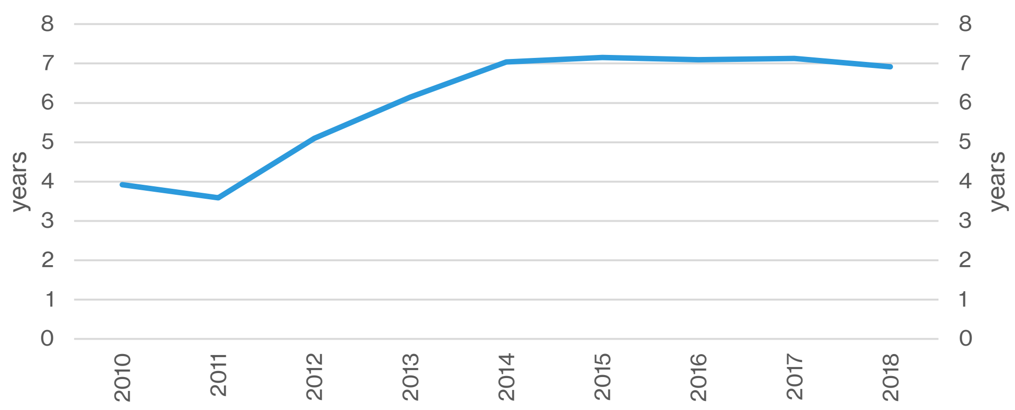
Appendix 1: data

Figure 2: debt equation data



Source: MF SR, SO SR, own calculations

Figure 3: average portfolio maturity



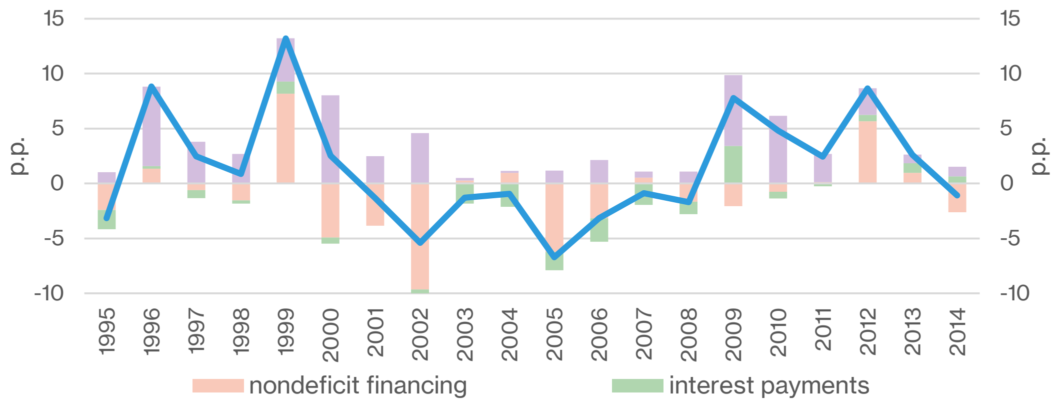
Source: IFP, own calculations

Appendix 2: growth of public debt

We manipulate the debt equation to get the growth of the debt on the left hand side. This equation then enables us to calculate contributions of individual components to the change in the level of public debt:

$$d_t - d_{t-1} = \frac{i_t - g_t - p_t}{(1 + g_t)(1 + p_t)} d_{t-1} - ps_t + nondeffin_t$$

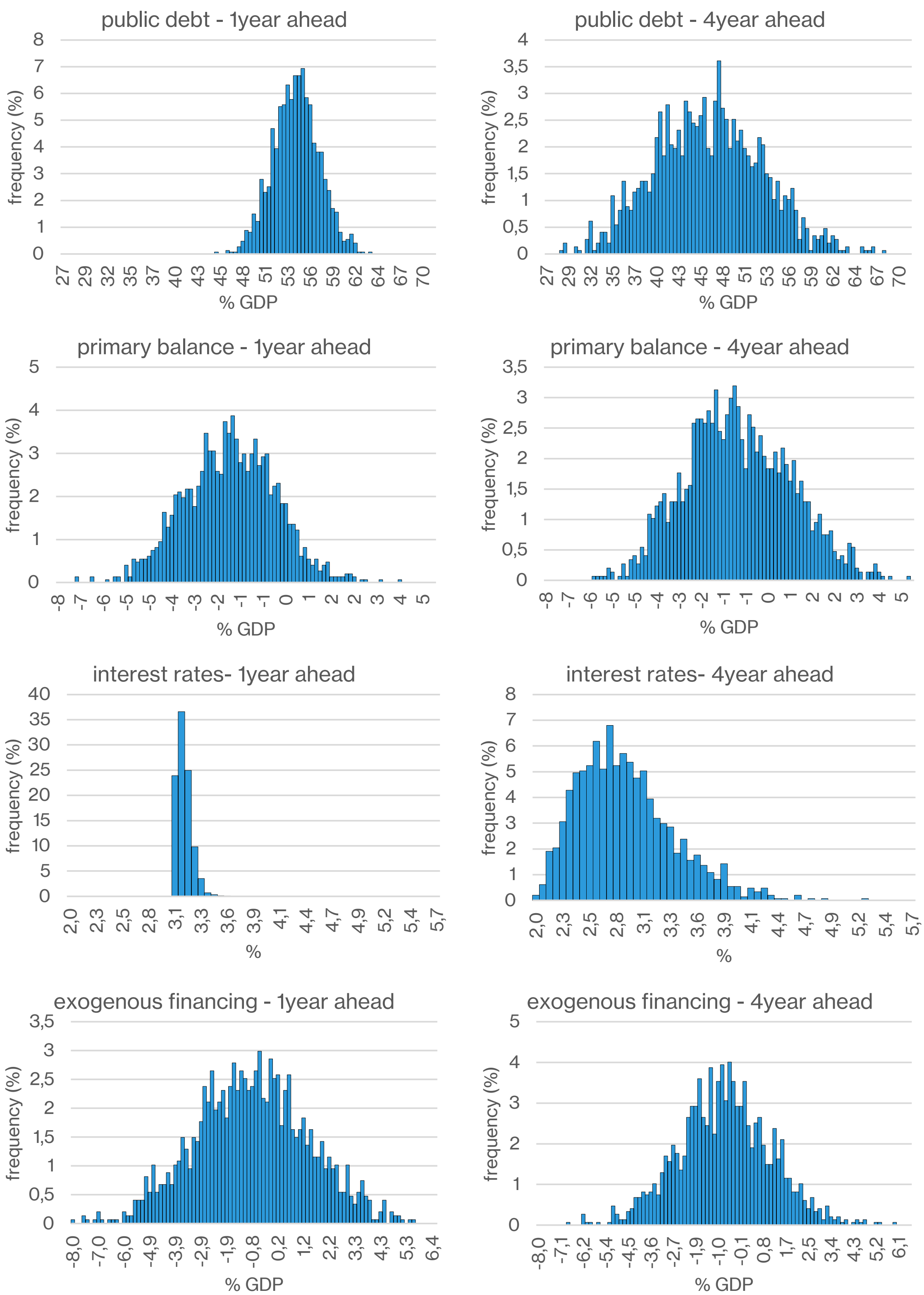
Figure 4: contributions to growth of public debt

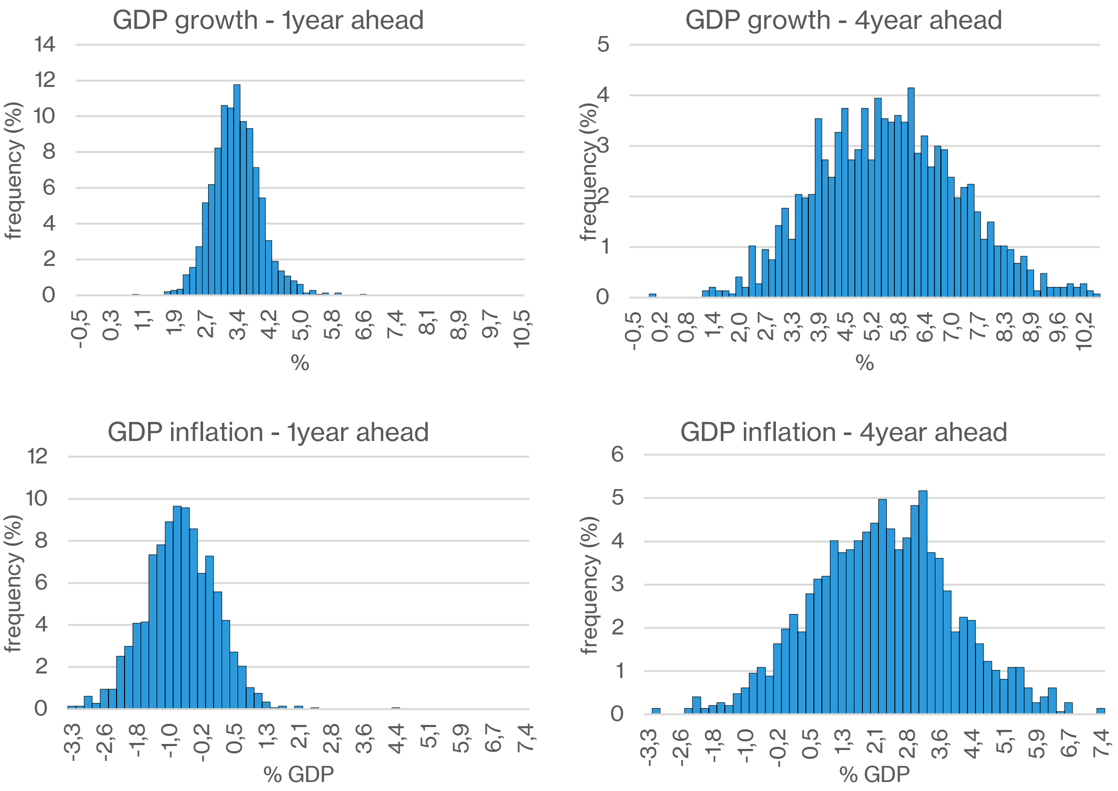


Source: MF SR, SO SR, own calculations

Appendix 3: detailed results

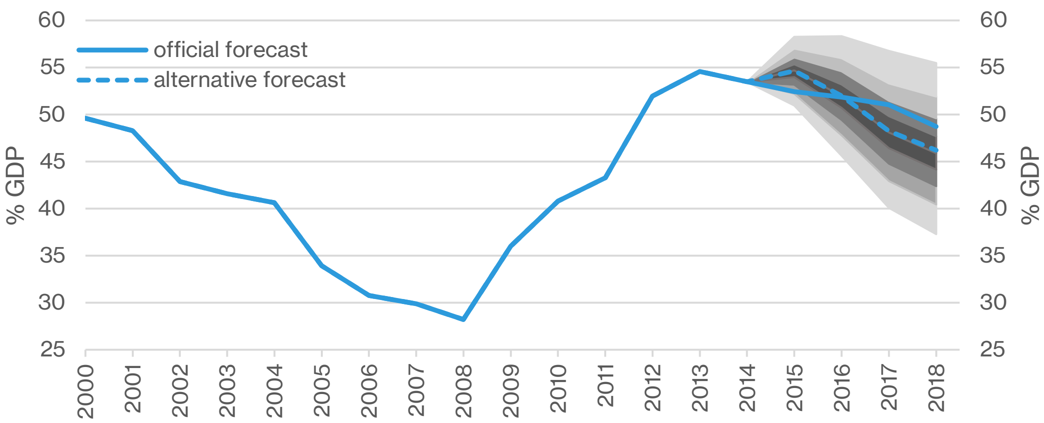
Figure 5: cross-section of forecasts





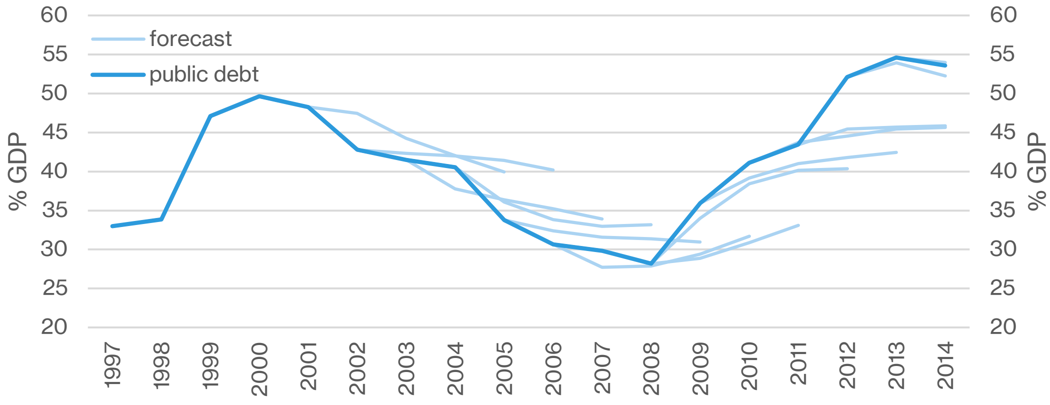
Source: own calculations

Figure 6: public debt forecast assuming normal distribution



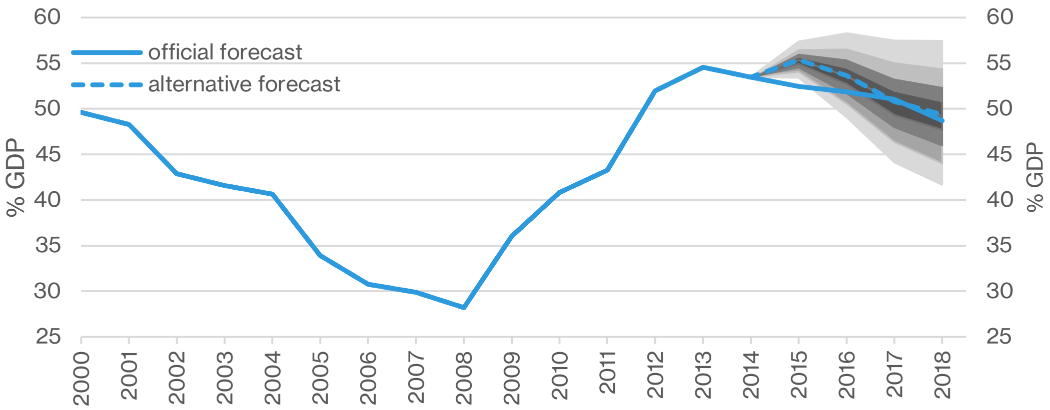
Source: own calculations

Figure 7: in-sample rolling forecasts of public debt



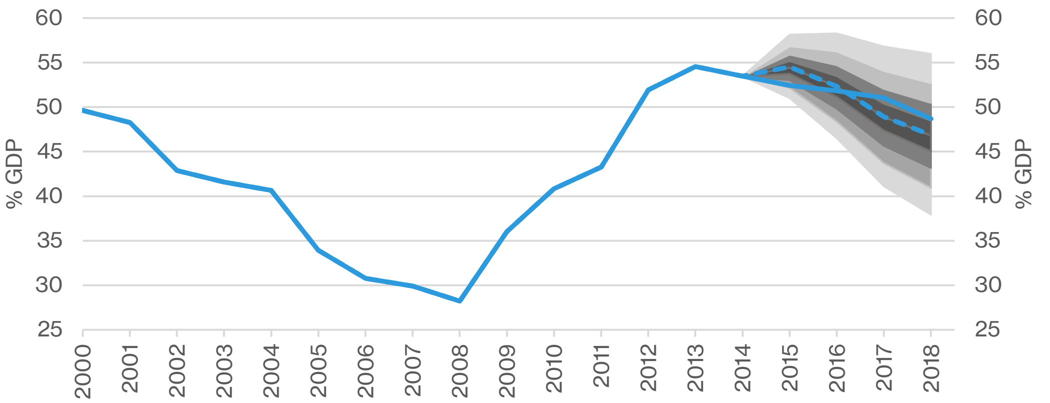
Source: own calculations

Figure 8: public debt forecast assuming no exogenous financing



Source: own calculations

Figure 9: forecast of public debt including negative interest rates



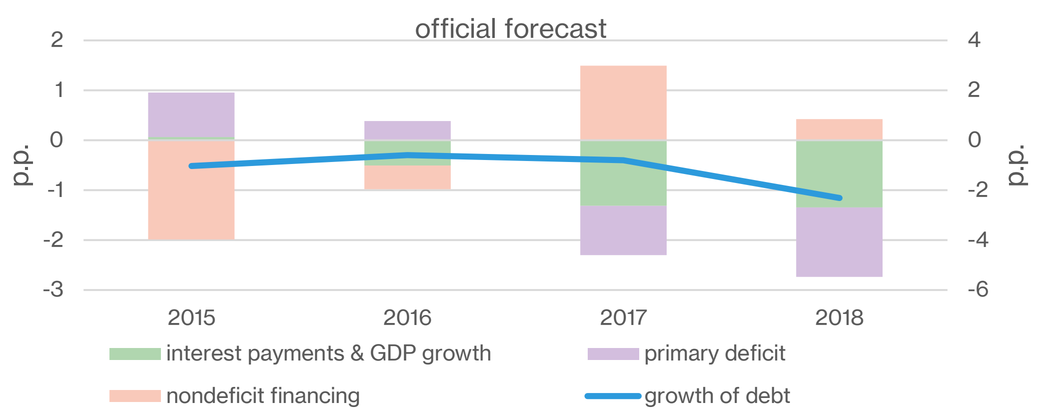
Source: IFP

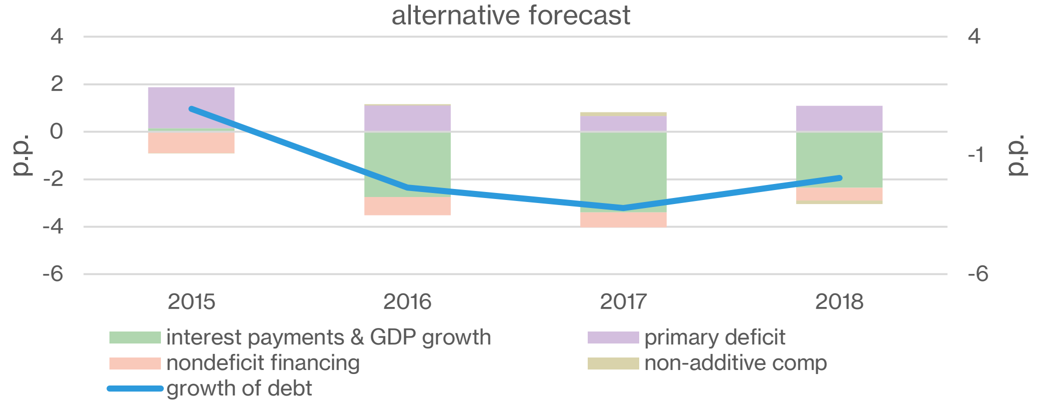
Figure 10: Fiscal reaction function – primary deficit

Source: own calculations

Appendix 4: factors driving the forecast

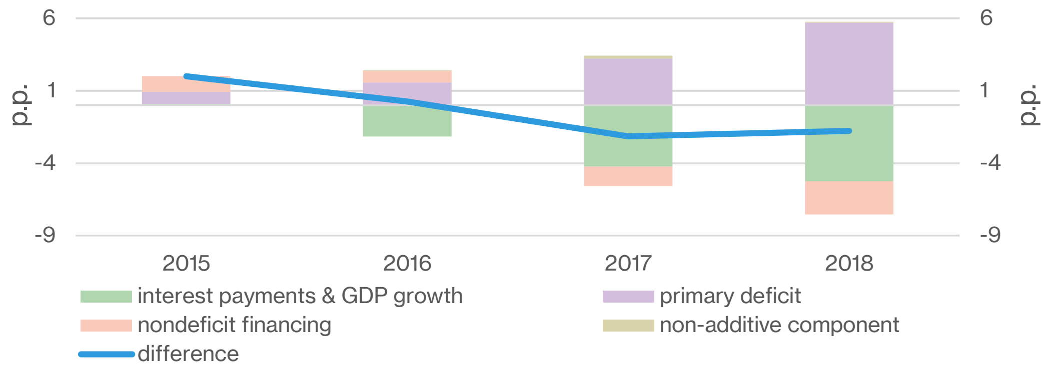
Figure 11: contributions to the forecasted growth of debt





Source: own calculations

Figure 12: factors driving the difference between the forecasted level of debt



Note: The contributions to the alternative forecast are based on the median values of the components which implies that the equation (1) does not hold exactly. The difference between the median alternative forecast and the level of debt obtained from equation (1) using the median values of the components is called non-additive components in the graph.

Source: own calculations